

Math And AI 4 Girls Competition 2023 Challenge Problem Set

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Rules:

- You have unlimited time to solve the problems. You may work on them until the competition closes, on April 23, 2023
- You are allowed to use calculators, books and other aides, although they are not necessary to solve the problems.
- We operate using the honor system, so we trust you to work alone on the problems without seeking help from peers and adults.
- For each question, please show your work and explain your reasoning - additional points will be given to **CLEAR and CONCISE explanations.**
- Your answers may be handwritten or typed.
- Please mark your name and the question number at the top of each sheet.
- Please do not have multiple questions per page.
- Please submit your work as one single PDF or word document.

Scoring:

- There are 11 problems in the problem set.
- The first 10 are math / logic problems. Each of these problems is worth 5 points, for a total of 50 points.
- The last problem is an AI / CS problem (math-related). The question is worth a total of 10 points.

Reference Sheet:

- There is a Reference Sheet which includes possible geometric theorems that may be useful in the contest.
- A brief introduction to mathematical induction proof writing is included in the reference sheet.
- The reference sheet can be accessed here:

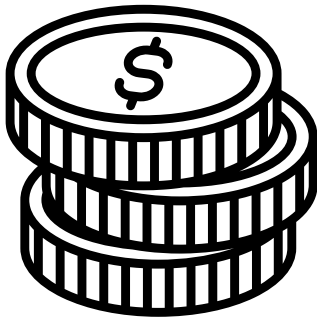
<https://drive.google.com/file/d/16YMF5-azFgKEQzFiehkleyL1fQbU1jc/view?usp=sharing>

PLEASE NOTE THAT THE TEST IS MEANT TO BE CHALLENGING, SO PLEASE DO NOT BE DISCOURAGED IF YOU CANNOT SOLVE ALL OF THE PROBLEMS. JUST DO YOUR BEST! AND REMEMBER, PARTIAL CREDIT MAY BE AWARDED FOR THE CORRECT SET-UP / REASONABLE IDEAS. GOOD LUCK!

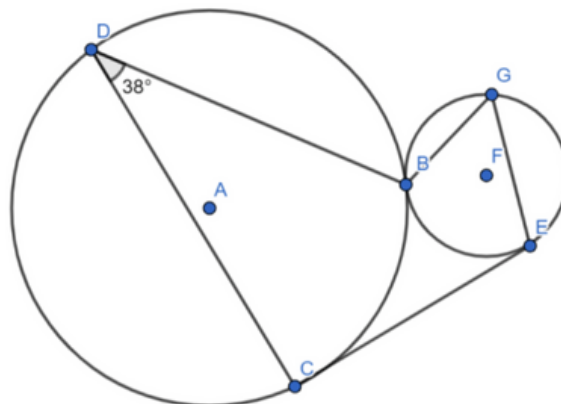
- Sophia wants to buy cookies that cost \$1.25 using as many coins as possible. She has dimes, nickels, and quarters with exactly 20 of each kind. What is the maximum number of coins Sophia can use to pay for her cookies?



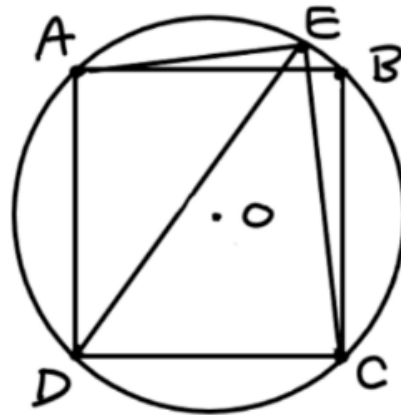
- Ivy organized a bake sale! She sold x ice cream bars, y pies, and z cookies, and it satisfies the equation $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{6}$. Find the triples of positive integers $x, y,$ and z , where $x \leq y \leq z$.



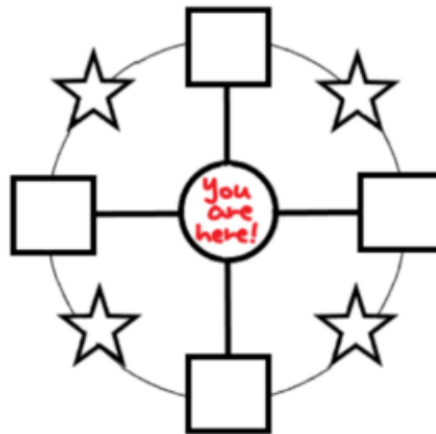
- Sam and Vaidehi have a coin tossing contest in which a fair coin is tossed 40 times. Sam will win if at least 21 consecutive heads show up, and Vaidehi will win if at least 21 consecutive tails show up. What's Sam's chance of winning the contest?
- There are 200 girls who are participating in the Math and AI 4 Girls Competition. A survey was conducted and the results showed that 168 girls loved math, 199 enjoyed participating in competitions, 100 liked earth science, and 157 liked computer science. At least how many girls like all four?
- Angie and Anika are heading from point X to point Y , with speeds of 65 meters per minute and 87 meters per minute, respectively. At the same time, Natalie starts jogging from point Y and jogs toward point X with a speed of 112 meters per minute. The distance between point X and point Y is 1504 meters. How long does it take for Natalie to be exactly in the middle of Angie and Anika?
- In the diagram as shown, $\angle CDB = 38^\circ$, $B, C,$ and E are points of tangency, and A and F are the centers of the two circles. What is the degree measure of $\angle BGE$?



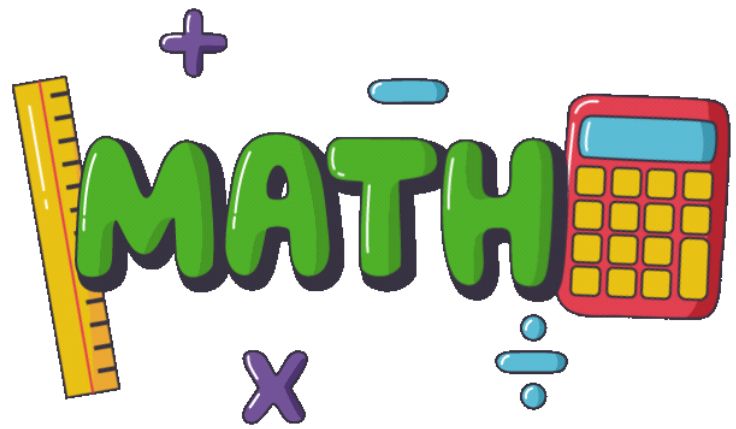
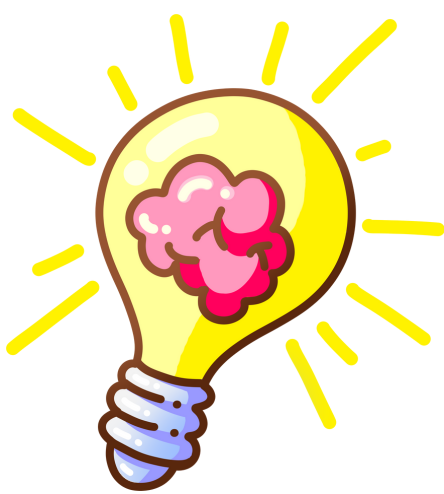
7. Towns A, B, C, D, and E, are positioned in a circle such that towns A, B, C, and D form a square. Town E is between towns A and B. Straight roads are built between the towns. If the road between towns A and E is 6 kilometers and the road between towns C and E is 8 kilometers, how long is the road between towns D and E?



8. You have been placed in the center of a maze. Each room is indicated by a shape and paths are indicated by lines. On each move you make, you randomly pick a path to take, leading you to a different room, independent of any previous moves. Once you reach a star-shaped room, you win (and stop making new moves). What is the expected number of moves you will make before winning?



9. There are 10 people standing in a circle. Some of them always tell the truth. Some of them always lie. The rest of them sometimes tell the truth and sometimes lie. There is at least 1 of each type of person. When asked if they were standing next to at least 1 truth-teller, all of them responded yes. When asked if they were standing next to only truth-tellers, every other person responded with yes and the rest responded with no. Find all possible configurations. (Rotations and reflections do not count as different configurations).



10. (Challenge !!) Prove that $\frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \dots + \frac{1}{n^3} < \frac{3}{2}$ for every positive integer n , such that $n \geq 1$.

11. In an m by n grid of characters, where each character is either 1 or 0, we can draw a rectangle by letting 1s represent squares inside the rectangle and 0s represent squares outside the rectangle. In the diagram below, the rectangle is outlined in red. However, sometimes, there isn't a perfect rectangle. Define the error to be the least number of squares

0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0	0	0	0
0	1	1	1	1	1	1	0	0	0	0
0	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

that are labeled incorrectly across all possible rectangles in the grid. Define an almost-rectangle to be a grid where the error is at most 4.

(a) Is the following grid an almost-rectangle?

0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	0	1	0	0	0	0
0	0	1	1	0	1	0	0	0	0	0
0	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

(b) Given an m by n grid of characters where each character is either 1 or 0, write pseudocode that checks whether or not the grid contains an almost-rectangle.