

Problem 2 (Geometry)

Calculate how many black pieces (pentagons) and white pieces (hexagons) are on a standard soccer ball.



Solution: We will use Euler's:

$$F + V - E = 2,$$

where F is the number of faces, V is the number of vertices, and E is the number of edges in the polyhedron.

Let B be the number of black (pentagonal) faces and W be the number of white (hexagonal) faces. We have that

$$F = B + W.$$

In addition, since each vertex is shared by 3 faces, and each pentagon has 5 vertices, and each hexagon has 6 vertices, we have

$$3V = 5B + 6W.$$

However, we also have

$$V = 5B.$$

Thus,

$$5B = 3W$$

Since each pentagon has 5 edges and each hexagon has 6, and each edge is shared by 2 faces, we have

$$2E = 5B + 6W.$$

Now, we solve the equations for B and W . Plugging in the values of F, V, E into Euler's formula, we get

$$B + W + 5B - \frac{5}{2}B - 3W = 2$$

$$\frac{7}{2}B = 2 + 2W$$

$$= 2 + \frac{10}{3}B$$

$$\implies \boxed{B = 12, \quad W = 20}$$