

Let

$$x = 100a + 10b + c \quad \text{and} \quad y = 100c + 10b + a,$$

where a, b, c are digits and $a, c \neq 0$ (since x, y are three-digit numbers).

Because $x + y$ is a multiple of 13,

$$x + y = (100a + 10b + c) + (100c + 10b + a) = 101(a + c) + 20b \equiv 0 \pmod{13}.$$

Also, x and y are multiples of 11. For a three-digit number $100a + 10b + c$, divisibility by 11 is equivalent to

$$a - b + c \equiv 0 \pmod{11}.$$

Since $a - b + c \in [-9, 18]$, the only possible multiples of 11 in this range are 0 and 11, so

$$a - b + c = 0 \text{ or } 11 \iff a + c = b \text{ or } b + 11.$$

Case 1: $a + c = b$. Then

$$101(a + c) + 20b = 101b + 20b = 121b \equiv 0 \pmod{13}.$$

But $121 \equiv 4 \pmod{13}$, so this implies $4b \equiv 0 \pmod{13}$, hence $b \equiv 0 \pmod{13}$. Thus $b = 0$. Then $a + c = b = 0$, impossible for three-digit numbers. So this case cannot occur.

Case 2: $a + c = b + 11$. Substitute into the 13-divisibility condition:

$$101(a + c) + 20b = 101(b + 11) + 20b = 121b + 1111 \equiv 0 \pmod{13}.$$

Reducing modulo 13,

$$121b + 1111 \equiv 4b + 6 \equiv 0 \pmod{13} \implies 4b \equiv 7 \pmod{13}.$$

The inverse of 4 modulo 13 is 10 (since $4 \cdot 10 = 40 \equiv 1 \pmod{13}$), so

$$b \equiv 7 \cdot 10 \equiv 70 \equiv 5 \pmod{13}.$$

As b is a digit, we get $b = 5$.

Next, using $x - y = 198$:

$$x - y = (100a + 10b + c) - (100c + 10b + a) = 99(a - c) = 198,$$

so

$$a - c = 2.$$

Also, from $a + c = b + 11$ and $b = 5$,

$$a + c = 16.$$

Solving

$$a - c = 2, \quad a + c = 16$$

gives $a = 9$ and $c = 7$. Therefore

$$x = 957, \quad y = 759, \quad x + y = 1716.$$