Math and AI 4 Girls 2026 Challenge Problem Set

Problems written by the Math and AI 4 Girls Problem Set Team

Rules

- You have **unlimited time** to solve the problems. You may work on them until the competition closes.
- You are allowed to use calculators, books, and other aids, although they are not necessary to solve the problems. You are **NOT permitted to use AI** on any part of the competition.
- We operate using the honor system, so we trust you to **work alone** on the problems without seeking help from peers or adults.
- For each question, please **show your work** and explain your reasoning. **Full credit** will **ONLY** be given to solutions with **CLEAR and CONCISE explanations**.
- Your answers may be **handwritten or typed**.
- Please do **not** have multiple solutions on one page.
- Please submit your work as **one single PDF or Word document.**

Scoring

- There are **sixteen problems** in the problem set, worth a total of **100 points**. The problems are arranged in a rough order of difficulty, but this ordering is only approximate, so you should feel free to skip around rather than work strictly in order.
- The first **fifteen problems** are pure mathematical problems. The last problem is a math-related **AI/CS** problem.
- The number of points for each problem is written next to the problem.

Please note that the test is meant to be **challenging**, so do not be discouraged if you cannot solve all the problems. Just do your **best**. Remember that **partial credit** may be awarded for correct setup or reasonable ideas. **Good luck!**

Problems

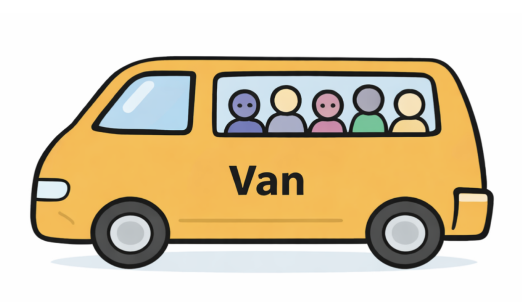
1. **[4 points]** Every week, Alice tells the truth on exactly two days and lies on the other five days. She lies on the same days every week. One week, Alice makes one statement daily on four consecutive days, in the given order:

   Statement 1: If I tell the truth tomorrow, then today is Saturday.
   Statement 2: I lie on Tuesdays and Saturdays.
   Statement 3: Yesterday, I lied.
   Statement 4: I will tell the truth exactly once in the next 3 days.

   On which two days does Alice tell the truth, and on which days does Alice make each of the statements? Find all possible solutions.

2. **[4 points]** Alyssa and Yara are looking at the MA4G merch store. They notice that if they both get a hoodie, Alyssa gets a t-shirt, and Yara gets a notebook, the total cost will come out to be \$115. They also notice that if they both get a t-shirt and notebook, and Yara gets a hoodie, the cost will be \$110, and if they both get a hoodie and t-shirt, the cost will be \$130. What is the price of a hoodie and notebook?

3. **[4 points]** A group of people are driving to a math competition in vans. If they use vans that seat 6 people each, there are 5 people left over. If they use vans that seat 7 people each, there are 6 people left over. In the end, they decided to use vans that seat 5 people, since no one was left over. What is the minimum number of people going to the math competition?



4. **[4 points]** A regular dodecagon with vertices $A_1, A_2, A_3, \ldots, A_{12}$ is inscribed in a circle with radius 1 and center M. Let G be the centroid of triangle $\triangle MA_1A_{11}$. Find the area of triangle $\triangle MA_4G$.

5. **[4 points]** Let a, b, c be the roots of the cubic polynomial

$$x^3 - 6x^2 + px - q = 0.$$

It is given that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1 \qquad \text{and} \qquad \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=3.$$

Find the value of q.

6. **[5 points]**

$$\prod_{x=0}^{2025}\left(\frac{x^3+12x^2+47x+60}{x^3+21x^2+146x+336}\right)=\left(\frac{0^3+12\cdot 0^2+47\cdot 0+60}{0^3+21\cdot 0^2+146\cdot 0+336}\right)\left(\frac{1^3+12\cdot 1^2+47\cdot 1+60}{1^3+21\cdot 1^2+146\cdot 1+336}\right)\cdots$$

$$\left(\frac{2025^3+12\cdot 2025^2+47\cdot 2025+60}{2025^3+21\cdot 2025^2+146\cdot 2025+336}\right)$$

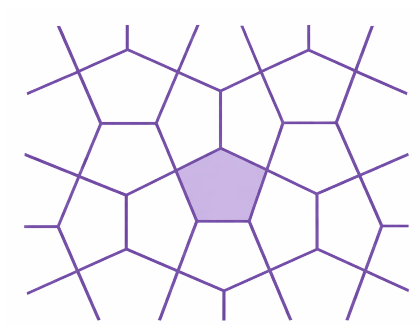
simplifies to a fraction of the form

$$\frac{a\cdot b^2\cdot c^3\cdot d^2\cdot e}{f\cdot g^2\cdot h^3\cdot i^2\cdot j},$$

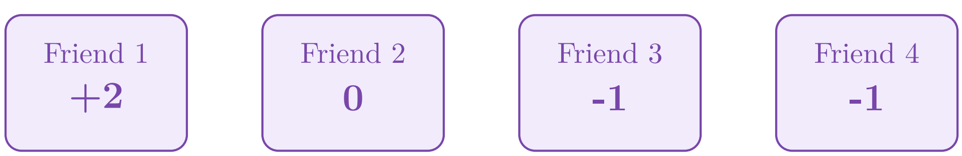
where a,b,c,d,e are consecutive positive integers and f,g,h,i,j are consecutive positive integers. Find

$$a+b+c+d+e+f+g+h+i+j.$$

7. **[5 points]** There is a specially made pentagonal floor tile shown below where the length of each side is 1. They can be used to pave the ground seamlessly as shown in the figure below. Find the area of each tile.



8. **[6 points]** Four friends decide to play a game. In the game, each friend can either win \$2, win \$1, break even, lose \$1, or lose \$2. What each friend gets is independent of the others. How many ways are there for the friends to collectively break even after all four have played the game?



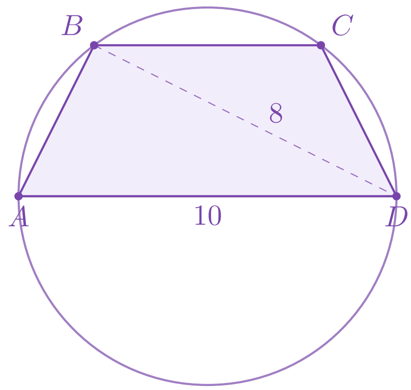
$$2+0-1-1=0$$

Example outcome where the friends collectively break even

9. **[6 points]** An arithmetic progression is a sequence of numbers in which the difference between consecutive terms is constant. One example is $\{1,3,5,7\}$, where the common difference is 2. In how many ways can the integers $\{1,2,\ldots,2026\}$ be colored red or blue such that no arithmetic progression of length 3 is entirely one color?

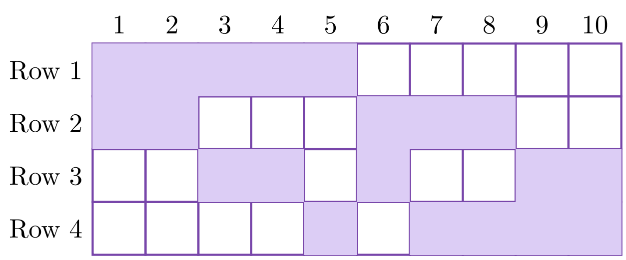
10. **[6 points]** Let x be the smallest positive integer with exactly 2026 divisors. What is the remainder when x is divided by 100?

11. [**7 points**] In the diagram shown, quadrilateral $ABCD$ is inscribed in a circle. Segment AD is a diameter of the circle with length 10, and $AD \parallel BC$. If $BD = 8$, find the area of quadrilateral $ABCD$.



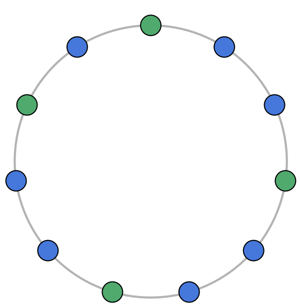
12. [**7 points**] Find the number of positive integers $n \leq 2025$ for which $\gcd(n, 2025)$ has exactly 3 positive divisors.

13. [**8 points**] In a class with 20 students, there are 40 seats arranged in 4 rows and 10 columns. How many ways are there for the students to occupy seats such that every row has 5 students and every column has 2 students? Two seatings are considered the same if the same seats are occupied, regardless of who sits in them.



Example seating arrangement with 5 students in each row and 2 in each column

14. [**9 points**] Joyce wants to make a bracelet with 11 beads. She has many blue and green beads she can use. If no two green beads can be adjacent on the bracelet, and two bracelets are considered indistinguishable if one can be obtained from the other by rotation, how many distinct bracelets can Joyce make?



Example of an 11-bead bracelet with no two green beads adjacent

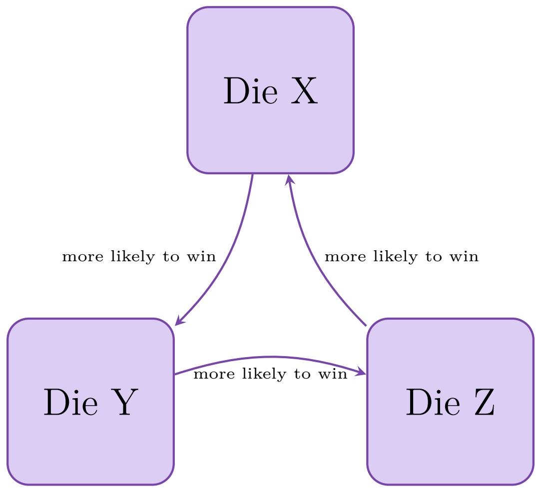
15. [**10 points**] Let $A, B > 0$. Define, for $0 \leq \theta \leq 2\pi$,

$$f(\theta) = \frac{(A\sin\theta - B\cos\theta)^3\,(A\cos\theta + B\sin\theta)^2}{(A^2 + B^2)^{5/2}}.$$

Find the maximum value of $f(\theta)$.

16. **[11 points]** We have three six-sided dice, Die A, B, and C. Given an arrangement of numbers on Die A and Die B (for example, A is $1, 2, 3, 4, 5, 6$ and B is $2, 3, 4, 5, 6, 7$), write pseudocode to determine whether it is possible to assign numbers to Die C such that the set of dice is non-transitive. If so, output "yes." If not, output "no." The numbers on the faces of Die A and Die B are integers from 1 to 8, inclusive, not necessarily distinct, and the numbers on the faces of Die C must also be integers from 1 to 8, inclusive.

Non-transitive means that "Die X" is more likely to win against "Die Y," "Die Y" is more likely to win against "Die Z," and "Die Z" is more likely to win against "Die X." In other words, no two dice tie and no one die is best. Assume that the given dice, Die A and Die B, do not tie.



Example of a non-transitive cycle: $X \to Y \to Z \to X$