2024 Math and AI 4 Girls Problem Set

Math and AI 4 Girls Team

Rules

- You have unlimited time to solve these problems, and you may work on them until the competition closes.
- You are allowed to use calculators, books and other aides, although they are not necessary to solve the problems.
- We operate using the honor system, so we trust you to work alone on the problems without seeking help from peers and adults.
- For each question, please show your work and explain your reasoning - full credit will ONLY be given to questions with CLEAR and CONCISE explanations.
- Your answers may be handwritten or typed.
- Please do not have multiple solutions on one page.
- Please submit your work as one single PDF or word document.

Scoring

- There are 12 problems in the problem set, worth a total of 75 points.
- The first 11 are math/logic problems. Each of these are worth 6 points, for a total of 66 points.
- The last problem is an AI/CS problem (math related). This problem is worth 9 points.

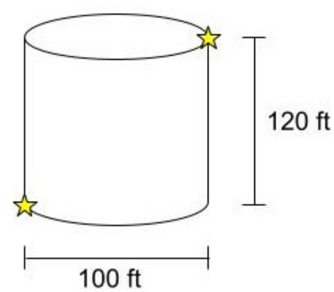
Please note that the test is meant to be challenging, so do not be discouraged if you cannot solve all the problems. Just do your best! And remember, partial credit may be awarded for the correct setup / reasonable ideas. Good luck!

Problems

1. How many ways can 12 identical pieces of candies be distributed to 4 students if every student gets at least 1 piece of candy and nobody can get more than 6 pieces?

2. Find the real value(s) of x such that $(x+5)(x+6)(x+7)(x+8) = 5040$.

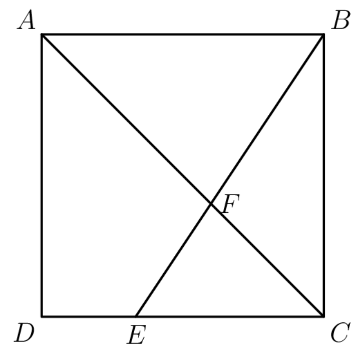
3. Tina is climbing a castle with the shape of a cylinder. The castle's two bases have diameter 100 feet and the height of the castle is 120 feet. Tina is currently at a point on the bottom base of the castle and wants to get to the opposite side of the top base. Given that Tina can only climb along the outside surface of the castle, what is the length of the shortest path that will get Tina there? Express your answer in feet.



4. In their respective bases, $abab_6 = bbc0_8$ where a, b, and c are nonnegative integers and $c = 2b$. Find the base 10 value of $abab_6$.

5. A person is painting her room. After painting half her room, she gets tired and takes a 15 minute break. After her break, she paints her room at 75% of her original pace. If the time from when she started to when she finished her break is the same as the time she spent painting after her break, how much time did she spend in total, including the break?

6. On square $ABCD$, $AB = \frac{3}{2} \cdot EC$. The intersection of segments BE and AC is F. The area of triangle AFB is 27. Find the area of square $ABCD$.

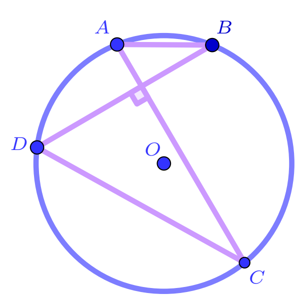


7. Vaidehi is having a garage sale. She has 504 dance costumes she is selling. Each person buys a different positive integer number of them, and the number of costumes each person buys end up being a sequence of consecutive integers. What is the greatest number of people who could buy her dance costumes?

8. Find the product of all positive integers x such that $\frac{2x^3-7x^2-2x+28}{2x-7}$ is a positive integer.

9. On an empty 8x8 chessboard, Sophia places a king and a queen on two distinct squares at random. Assuming the two pieces are different colors, what is the probability that the king is in check? (In other words, what is the probability that the queen—who can move any number of spaces horizontally, vertically, or diagonally—can move to the square the king is on in one move?)

10. In unit circle O, chords $\overline{AB}$ and $\overline{CD}$ have lengths x and $3x$ respectively, and $\overline{AC}$ is perpendicular to $\overline{BD}$. Find x.



11. On a remote island, all inhabitants are either mathematicians or machines. Mathematicians always tell the truth, and machines always tell lies. Equipped with this information, an adventurer wants to figure out which of her friends (all of which are inhabitants of said remote island) ate the last cookie. Their statements are as follows:

    Alice: I didn't eat the cookie.
    Bob: Alice is lying.
    Charlie: I ate the cookie.
    Dave: Both Alice and Emily are telling the truth.
    Emily: A mathematician ate the cookie.
    Fred: Bob is telling the truth.

    Given that exactly two of the adventurer's friends are mathematicians, determine whether each friend is a mathematician or a machine, and find the cookie-eating culprit.

12. We have 10 points, each of which is either red or blue. Given an uncategorized point, we can categorize by looking at the closest point and classifying it to the same color.
    (a) Write pseudocode that, given 10 categorized points and an uncategorized point, finds which color the point should be using the algorithm described above.
    (b) Instead of looking at just the singular closest point, we can instead look at the three closest points and take the majority color. How would your pseudocode from above have to be revised for this algorithm?