

First of all, we can form a one to one correspondence! The arrangement of balls to buckets can be paralleled to strings of R, G, and B where the ball at each position gets placed into the corresponding bucket. For instance, RGRBBGB means that balls 1 and 3 go into the red bucket, balls 2 and 6 go into the green bucket, and balls 3, 4, and 7 go into the blue bucket.

Now, the question is essentially asking for the number of R, G, B strings of length 7 where no two Rs, no two Gs, and no three Bs are consecutive.

Next, we set  $r_n$  as the number of  $n$  length strings that end in R,  $g_n$  as the number that end in G, and  $b_n$  as the number that end in B.

We can now use recursion! If the last letter in a string is R, then the second-last can be B or G. This means that  $r_n = g_{n-1} + b_{n-1}$ , since  $r_n$  consists of strings with length  $n - 1$  ending in B or G, with an R stuck at the end. Similarly,  $g_n = r_{n-1} + b_{n-1}$ . Lastly,  $b_n = r_{n-1} + g_{n-1} + r_{n-2} + g_{n-2}$ , since the second to last letter can be R, G, or B. If it's B, then the third to last letter can be R or G.

Now, we calculate the base cases!  $r_1 = g_1 = b_1 = 1$ , since the strings are R, G, and B respectively.  $r_2 = 2$  (GR, BR),  $g_2 = 2$  (RG, BG), and  $b_2 = 3$  (BB, RB, GB). Using the recursive relationship we obtained earlier, we get that  $r_7 = g_7 = 129$ , and  $b_7 = 164$ , so the total number of length 7 strings is 422.

Lastly, we need to subtract the overcounted cases where the string only consists of Rs and Gs. There are only 2 strings that satisfy the condition: RGRGRGR and GRGRGRG. Therefore, our answer is  $\boxed{420}$ .