

## Solution.

Define the height of the pyramid as  $H$ .

First shave: The plane passes through the vertex and points  $(0, 1)$  and  $(1, 0)$ , cutting off a slanted pyramid with a right isosceles triangle base of area

$$A_1 = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}.$$

Its volume is

$$V_1 = \frac{1}{3} A_1 H = \frac{1}{3} \cdot \frac{1}{2} H = \frac{H}{6}.$$

Second shave: The plane passes through the vertex and points  $(0, 2)$  and  $(2, 0)$ , forming a larger slanted pyramid of base area

$$A_2 = \frac{1}{2} \cdot 2 \cdot 2 = 2.$$

Subtracting the first piece gives the volume of the second shaved piece:

$$V_2 = \frac{1}{3} A_2 H - V_1 = \frac{1}{3} \cdot 2H - \frac{H}{6} = \frac{H}{2}.$$

Volume ratio:

$$\frac{V_1}{V_2} = \frac{\frac{H}{6}}{\frac{H}{2}} = \frac{1}{3}.$$