Solution 1: Factor

By the Rational Root Theorem, the possible rational roots of this polynomial are $\pm 2, \pm 1, \pm \frac{1}{2}$. We can test these few options and find that the polynomial factors into $(x+2)(2x-1)(x^2-6x-1)=0$. For $x=-2$ or $x=\frac{1}{2}$, we have $x-\frac{1}{x}=\boxed{-\frac{3}{2}}$. The quadratic formula on $(x^2-6x-1)=0$ gives us $x=3\pm\sqrt{10}$ for which $x-\frac{1}{x}=\boxed{6}$.

Solution 2: Divide by x^2

We have

$$\begin{aligned} 2x^4-9x^3-22x^2+9x+2 &= x^2\left(2x^2-9x-22+\frac{9}{x}+\frac{2}{x^2}\right) \\ &= x^2\left[2\left(x^2+\frac{1}{x^2}\right)-9\left(x-\frac{1}{x}\right)-22\right]=0. \end{aligned}$$

Quickly checking, we see that $x=0$ is not a solution so we must have $2\left(x^2+\frac{1}{x^2}\right)-9\left(x-\frac{1}{x}\right)-22=0$. Let $a=x-\frac{1}{x}$, which means $a^2=x^2+\frac{1}{x^2}-2$ so we have

$$\begin{aligned} 2(a^2+2)-9a-22 &= 2a^2-9a-18 \\ &= (2a+3)(a-6)=0. \end{aligned}$$

This gives us $a=\boxed{-\frac{3}{2},6}$.