

# Math And AI 4 Girls Competition 2022 Challenge Problem Set

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## Rules:

- You have unlimited time to solve the problems. You may work on them until the competition closes, on April 24th, 2022
- You are allowed to use calculators, books and other aides, although they are not necessary to solve the problems.
- We operate using the honor system, so we trust you to work alone on the problems without seeking help from peers and adults.
- For each question, please show your work and explain your reasoning - full credit will ONLY be given to questions with **CLEAR and CONCISE** explanations.
- Your answers may be handwritten or typed.
- Please mark your entry code (see instructions on application portal) and the question number at the top of each sheet.
- Please do not have multiple solutions on one page.
- Please submit your work as one single PDF or word document.

## Scoring:

- There are 12 math and AI problems in the problem set.
- Each question is also worth 5 points, for a total of 60 points.

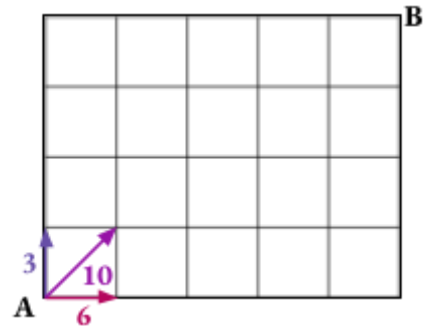
PLEASE NOTE THAT THE TEST IS MEANT TO BE CHALLENGING, SO PLEASE DO NOT BE DISCOURAGED IF YOU CANNOT SOLVE ALL THE PROBLEMS. JUST DO YOUR BEST! AND REMEMBER, PARTIAL CREDIT MAY BE AWARDED FOR THE CORRECT SET-UP / REASONABLE IDEAS.

1. An animal shelter has a total of 20 cats, dogs, and parrots. There are at least twice as many cats as dogs and at least 3 times as many parrots as dogs. Find the maximum number of dogs they could have.
2. There exist three boxes of books: A, B, and C. Boxes A and B weigh a total of 10 pounds. Boxes B and C weigh a total of 9 pounds. Boxes C and A weigh a total of 11 pounds. Find the weight of each box.



3. An ant starts at point A and wants to get to point B. It can:
  - Walk up a unit, which takes 3 seconds
  - Walk to the right by one unit, which takes 6 seconds,
  - Walk diagonally, which takes 10 seconds.

What is the minimum amount of time it can use to travel from point A to point B?



4. Find nonzero digits A, B, and C that make the following equation true:

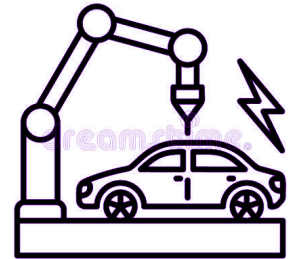
$$\begin{array}{r} 3A \\ \times B9 \\ \hline C08 \end{array}$$

5. Every second, an elevator either has a 50% chance of going up and a 50% chance of going down. If the elevator starts at floor 15, what is the probability that, after 5 seconds, the elevator is at floor 18?
6. A three-digit number is called special if its digits form an increasing arithmetic sequence—that is, the difference between the first and second digit is the same as the difference between the second and third digit, and each digit is **larger** than the one to the left of it. For example, 567 is special, but 236, 111, 048, and 975 are not. How many three-digit special numbers exist?

7. How many triangles are in the figure below?



8. A factory produces cars that are safe 80% of the time and uses a machine to test whether each car is safe. The machine marks safe cars incorrectly 10% of the time and unsafe cars incorrectly 40% of the time. If a car is marked as safe by the machine, what is the probability that it is actually unsafe?



9. A funky 6-sided die with faces labeled 1, 2, 3, 4, 5, and 6 has a  $\frac{1}{12}$  chance of landing on each face and a  $\frac{1}{24}$  chance of landing on each edge. When Kira rolls the die, if it lands on a face, she records the number on top. If she lands on an edge, she records the sum of the two numbers on top. If Kira rolled the die infinitely many times and averaged the result, what is the expected value of the number recorded?

### What are Number Bases?

We're used to doing things in base 10 (the decimal system). Think of the number 234 in **base 10**:

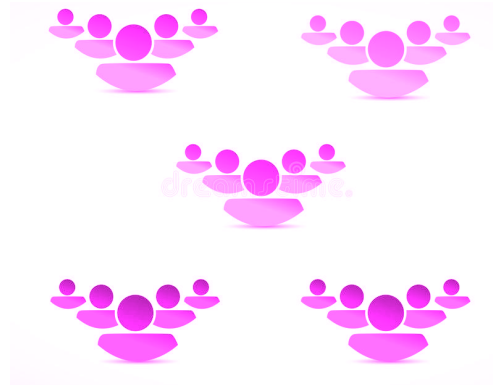
- The 4, which is in the units place, represents 4.
- The 3, in the tens place, represents  $3 \times 10 = 30$ .
- The 2, in the hundreds place, represents  $2 \times 10^2 = 200$ .

But what if we had a different base? If we had 234 in **base 9**, it would represent  $2 \times 9^2 + 3 \times 9 + 4$ .

Note: If we have a number written in base  $a$ , then each of its digits must be less than  $a$ . For example, in base 6, we can't have the digit 7. This rule ensures that there is only one way to write each number. Otherwise, in base 6, we would have 2 ways of writing 7 (both 7 and 11).

10. The aliens that live on the distant planet of Zorblox use a different number base from us! In binary, or base 2, Ivy's favorite number is 1111100110. However, on Zorblox, her favorite number is stored as 5616. What is Ivy's favorite number in base 10? What number base do the Zorbloxians use?

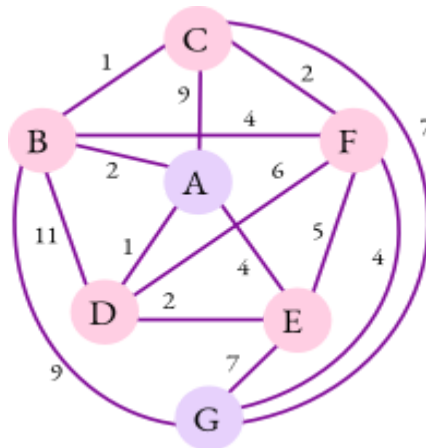
11. Suppose you have 4 teams: A, B, C, and D. Each team has a unique skill level, and in a game between any two teams, the team with the higher skill level always wins.
- What is the **minimum** number of games needed to ensure that you know the teams' rankings? Write which teams compete in each 1v1 match.
  - Team E has arrived, and its players are eager to compete! Now, with 5 teams, what is the minimum number of games needed to ensure that you know their rankings?



12. You are at city A, and your goal is to find the shortest route to city G!

You may pass through cities B, C, D, E, and F.

Each line connecting two cities has a number, which represents the number of days it takes to go between those two cities. Now, how many days (minimum) does it take to go from A to G, and what path do you take?<sup>1</sup>



<sup>1</sup> Roads are not drawn to scale