Number Theory

Find the smallest positive integer n such that:

$$n \equiv 2 \pmod 9$$
$$n \equiv 5 \pmod{11}$$
$$n \equiv 7 \pmod{13}$$

From the first two statements, we get that $n = 2 + 9k \equiv 5 \pmod{11}$, and the smallest possible value of n is now $n \equiv 2 + 9(4) = 38 \pmod{99}$

Now adding in the last statement, we know that $n = 38 + 99t \equiv 7 \pmod{13}$. Reducing the equation mod 13, we get $n = 12 + 8t \equiv 7 \pmod{13}$. Since the inverse of $8 \pmod{13}$ is 5, $t \equiv 40 \equiv 1 \pmod{13}$. Plugging this in, $n = 38 + 99(1) = \boxed{137}$