Math and AI 4 Girls 2025 Problem Set

Math and AI 4 Girls Problem Set Team

March 2025

1 Rules

- You have unlimited time to solve the problems. You may work on them until the competition closes.

- You are allowed to use calculators, books and other aides, although they are not necessary to solve the problems.

- We operate using the honor system, so we trust you to work alone on the problems without seeking help from peers and adults.

- For each question, please show your work and explain your reasoning - full credit will ONLY be given to questions with CLEAR and CONCISE explanations.

- Your answers may be handwritten or typed.

- Please do not have multiple solutions on one page.

- Please submit your work as one single PDF or word document.

2 Scoring

- There are thirteen problems in the problem set, worth a total of 100 points, which will then be scaled. The problems are arranged in rough order of difficulty.

- The first eleven problems are pure mathematical problems. The number of points for each problem is written next to the problem.

- The last two problems are a logic problem and a math-related AI/CS problem. These problems are worth nine points and thirteen points, respectively.

Please note that the test is meant to be challenging, so do not be discouraged if you cannot solve all the problems. Just do your best! And remember, partial credit may be awarded for the correct setup or reasonable ideas. Good luck!

3   Problems

1. This year, my age is my brother's age cubed. Next year, my age will be my brother's age squared. How old am I? Our ages are positive. (4 points)

2. In rectangle $ABCD$, point E is on CD such that $\angle AEB$ is a right angle. Given that $BC = 4$ and $EC = 3$, what is the area of $ABCD$? (5 points)

3. Mia and Noah are painters working on painting a large mural. Mia can paint the entire mural by herself in 5 hours, while Noah can paint it in 7 hours. When they work together, they can collaborate effectively, which increases their total efficiency by an extra 4 square meters per hour. Working together, they complete the mural in 2.5 hours. How many square meters is the mural? (5 points)

4. The Math and AI 4 Girls Problem Set team travels to a planet where the number system is based on the digits 1 through 7 inclusive. They need to catch a train, but the train numbers are special: they must contain the digit 4, the digit 5, or both. If the team considers only the 343 smallest positive integers in this number system, how many train numbers will meet this requirement? (6 points)

5. A certain function f has the following properties:

   1. $f(3x) = 3f(x)$ for all positive real values of x,

   2. $f(x) = 1 - |x - 2|$ for $1 \le x \le 3$.

Find $f(2025)$. (7 points)

6. Alice baked 30 identical cookies for Bob, Carol, and David to share. Bob wants at least two cookies, Carol wants at least three cookies, and David wants at least five cookies. How many ways are there to distribute the cookies among Bob, Carol, and David? (7 points)

7. Find the number of ordered pairs of nonnegative integer solutions (x, y) to the equation $20x + 25y = 2025$. (7 points)

8. Triangle ABC has point D on BC and point E on AC such that $BD : DC = 3 : 1$ and $AE : EC = 2 : 5$. Let P be the intersection of AD and BE. Find $BP : PE$, expressed as a common fraction in simplest form. (8 points)

9. Consider a 2×6 rectangular grid with the bottom-left 1×1 square missing. How many ways are there to completely tile this grid with non-overlapping 1×2, 2×1, and 1×1 pieces? (9 points)
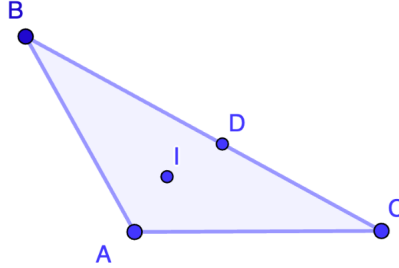
10. Given the polynomial

$$Q(x) = x^3 - 3x^2 + 100x - 10,$$

let its roots be r_1, r_2, and r_3. Compute the value of:

$$\frac{1}{r_1(r_1 - 1)} + \frac{1}{r_2(r_2 - 1)} + \frac{1}{r_3(r_3 - 1)}.$$

(10 points)

11. In $\triangle ABC$, $BC > BA$. Point D is chosen on $\overline{BC}$ such that $BA = BD$, and point I is the incenter of $\triangle ABC$. Given that $DC + AC = \sqrt{3} \cdot IC$, find the measure of $\angle ACB$ (in degrees). (10 points)

12. Alice and Bob are each given a distinct one-digit positive integer. They both know this but are unaware of each other's number. They are told that among the two numbers, there is at least one perfect square, among the two numbers, there is a number divisible by 3, and the two numbers are not relatively prime. Initially, neither Alice nor Bob can determine the other's number. However, upon realizing this mutual uncertainty, they deduce each other's number. What is the product of their numbers? Assume Alice and Bob are perfectly logical. (9 points)

13. You are given a random number generator that can generate a uniform random number between a user-designated **start** and **end** value. (Note: uniform means that each point has precisely the same probability of being chosen.) You can choose new **start** and **end** values each time you use the random number generator. You want to generate a uniform random point inside a unit circle centered at the origin. (13 points)

    a. What is wrong with the algorithm on the next page? (This article may be helpful if you're unfamiliar with trigonometry.)

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Let radius be a random number between 0 and 1
radius represents the distance between the origin and the point

Let angle be a random number between 0 and 2*pi
angle represents the angle between the x-axis and the line from the
origin to the point

To calculate the x and y coordinates of our random points:
     Let xCoord = radius * cos(angle)
     Let yCoord = radius * sin(angle)
Our random point then is at the point (xCoord, yCoord)
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b. Write pseudocode for an algorithm that successfully generates a uniform random point inside the circle.